Concepts for Composite System Reliability Applied to the Two-Area Planning Problem

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Abstract:

A new algorithm to compute the probability density function of the flows over a transmission tie between two areas is developed. This algorithm is integrated with composite system reliability concepts to compute the probability density function of load curtailments for each area. Penalties in terms of increased fuel costs due to departures from the optimal economic dispatch forced by tie capacity restrictions are also computed. Extrapolation of the concepts presented to the multi-area problem is briefly discussed.

1.0 Introduction

The function of an electrical power system is to supply the customer electricity demand or load. Accordingly, the most directly meaningful way to express the system reliability is in terms of load which needs to be curtailed because of equipment failure or because of operating action to prevent failure of overloaded equipment.

Within the system planning context, the load curtailment implied by a given generation and transmission expansion alternative, can be thought of in terms of five components, namely, curtailment due to:

[1] Generation equipment failures and capacity shortages
[2] Load forecast uncertainty
[3] Positioning of transmission elements with respect to generation and load
[4] Capacity limitation of transmission elements
[5] Failure of transmission elements.

To date, the established generation system reliability evaluation techniques(1) permit evaluation of system load curtailments due to items [1] and [2] above. Current development efforts in the area of composite system reliability appear to be concentrating largely on item [5] with implicit limited consideration of items [3] and [4] under transmission failure conditions. The explicit treatment of items [3] and [4] is being addressed to some extent by the current effort in the area of probabilistic load flow techniques(2-4). However, at this time, these techniques are not being viewed as reliability techniques.

The purposes of this paper is to present a modelling technique which allows unified evaluation of all of the above load curtailment components. The implementation of the technique involves the following steps:

Step 1:
Computation of the joint probability density function of power flows on all lines with recognition of optimal economic dispatch. This function is computed considering all the possible load and generation states under the assumption of no transmission failures.

Step 2:
Minimization or elimination of line overloads and undervoltages present in the load flow states computed in Step 1 as far as permitted by generation rescheduling. This results in less economical dispatches and thus in a fuel cost penalty due to transmission limitations.

Step 3:
Computation of load curtailments for each of the load flow states derived in Step 2 to eliminate overloads and undervoltages which could not be eliminated by further modification of generation dispatch.

Step 4:
The load flow states derived in Step 3 would have all flows and voltages at acceptable levels. The further load curtailments and/or fuel cost penalties which can be incurred as a result of transmission failure, are computed in this step.

The above computational steps will be illustrated below in details for a two area problem neglecting the effects of undervoltages.

2.0 Computation of Probability Density Function of Line Flows

The probability density function of the power flows on transmission lines results from:

[a] The probability and capacity value of the states in which the generation system can reside because of unit failures
[b] The probability and value of the loads in the two areas
[c] The probability of transmission line failures.

In this section, the computational algorithm will be presented for a two-area problem assuming a single load level and a completely reliable tie. The recognition of multiple load levels and of tie failures will be described in Sections 6.0 and 7.0.

2.1 Illustration of the Concepts Used

Under the assumption of a single load level in each of the two areas and a completely reliable tie, various flows will exist with various probabilities as a result of the states in which the generation in the two areas will reside due to unit failures.

In a practical-size system the total number of all the possible generating states is very large. For example a generation system consisting of 50 units can exist in 2^{50} possible states. This assumes that the units fail independently and that each can reside in only two states, say, "up" and "down". Consequently, computation of tie flow probabilities by explicit enumeration of all the possible generating states would result in computer time requirements far too large to be practical. This is one of the major difficulties in applying the probabilistic load flow techniques based on Borkowska paper(2) to the solution of large systems. This problem is resolved by reducing the total number of all the possible generation states into a smaller number of equivalent ones.

Because the number of all the possible generation states is too large for explicit enumeration, the state reduction process needs to be recursive as, for example, the one used to build the capacity outage probability table for the computation of the Loss of

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Load Probability (LOLP) index. The state reduction algorithm consists of two main steps, namely:

[1] Compute partial load flow states by adding units one at a time.
[2] After each unit addition, states whose load flow values are equal or similar are combined. This combination proceeds until the maximum number of states which can be handled is reached.

2.2 Computation of Partial Flows

A prerequisite to be able to compute correctly new partial flows after each unit addition is that flows need to be expressed as a linear combination of generating unit loadings and of the load to be supplied. It is easy to show that the DC approximation to the AC load flow problem implies the following linear relationship between line flows and nodal real power injections.

$$[F_e] = [H] \; [P] \qquad (1)$$

where:

$[F_e]$ = vector of line real power flows.

$[H]$ = an exn matrix where e is the number of lines and n is the number of nodes. The element H_{ij} represents the flow in line i due to a unity power injection at node j with all other node injections set to zero.

$[P]$ = vector of nodal power injections representing generation and load.

From (1) it follows that if:

$$[P] = \sum_{i=1}^{x} [P_i] \qquad (2)$$

then

$$[F_e] = [H][P_1] + [H] \, [P_2] + .. \, [H] \; [P_x] \qquad (3)$$

for any arbitrary value of $[P_i]$ and x.

Equation (3) suggests how to proceed with the calculation of partial flows after addition of each unit. This will now be illustrated with the two area system shown in Figure 1(a). The point to be illustrated is that the flow value associated with any of the final generation states can be computed correctly by superimposing partial flows computed after each unit addition. The generation of all the possible generation states is illustrated in Figure 1(b) using a conditional probability tree.

Figure 2(a) illustrates the computation of the tie flow using final State I in Figure 1(b) as an example. Figure 2(b) illustrates the computation of the same flow by superposition of flows computed after each unit addition. Note that economic dispatch has been recognized by adding the cheaper units first. Unit power production cost was assumed lowest with Unit 1 and highest with Unit 3.

Because of the economic dispatch, State I which has more generation capacity available than the load requires, results in a single set of unit loadings, namely, 10 MW on Unit 1 and 6 MW on Unit 3, since any other loading combination would be more expensive.

In Figure 2(b), after addition of Unit 1 in the up state, there was insufficient generation to meet the load. Therefore, before being able to solve the load flow, load needs to be curtailed to achieve generation load balance. The results shown were obtained assuming that load would be curtailed in the two areas in amounts proportional to the total loads in the respective areas. For the particular final state I, any other arbitrary load curtailment rule can be used without affecting the final answer. The reason for this is that State I has sufficient generation to meet all of the load in both areas. However, for a final state where there is not enough generation to meet all of the load, such as State K in Figure 1(b), the particular load curtailment rule used in the computation would be reflected in the final flow. For such states it is necessary to decide whether the load cutting rule used is consistent with existing or anticipated operating policies related to load curtailment due to generation shortages. In general any load curtailment operating policy related to generation shortages can be adhered to only to the extent permitted by the existing transmission capabilities. Assuming here that transmission limitations will never restrict application of any given load curtailment policy, then it can be shown that computation after each unit addition can be done assuming a load curtailment rule which simplifies computation. When the final flows are obtained, an additional flow modification is made by superimposing the flow resulting from a load injection vector which when superimposed on the final load curtailments computed, modifies these to be in accordance with the desired load curtailment policy.

2.3 Algorithm for the Computation of the Probability Density Function of Tie Flows

The computational steps involved are best described with an example. The system to be solved is illustrated in Figure 3 which gives all the relevant data including the most economic unit loading sequence. The results obtained at each computational step following unit additions are given in Tables I through VIII. These tables contain a description of the generation states which are generated after each unit addition. A generating state can be described in a number of ways depending on the answers sought. For example, in the capacity outage probability table for LOLP computation, generation states are described in terms of their capacity-on- outage values. In Table I through VIII, the generation states are described in terms of values relevant here, namely: tie flow FAB; state probability P; unsupplied loads UL_A and UL_B; the loading L and spare S on each of the units in the state. Note that the latter information is not required for the computation of tie flow probabilities, but will be required for the computation of fuel cost penalties as discussed in Sections 3.0.

Table I gives the results obtained after adding the cheapest unit, namely, Unit 1. State 1 in this table results with Unit 1 down. The generating capacity of this state is 0.0 MW and its probability is the same as the probability of Unit 1 being in its down state, namely, 0.1. Therefore the tie flow is 0.0 and the total original load remains unsupplied.

State 2 in Table I results with Unit 1 up. The generating capacity of this state is 200 MW and its probability is the same as the probability of Unit 1 being up, namely, 0.9. Therefore 200 MW of load can be supplied with the 100 MW of load remaining unsupplied shared between Area A and B in the amounts of 60 MW and 40 MW respectively. Of the total 200 MW of load supplied, 80 MW is in Area B. Since Unit 1 is in Area A, this results in a tie flow of 80 MW towards B.

Table II gives the results obtained after adding Unit 2. Four new states are generated. Two are obtained by combining Unit 2 in the down state with the two partial states in Table

I. Two more states are obtained by combining Unit 2 in the up state with the same two states in Table I.

It is noted that with Unit 2 down, the first two states in Table II are exactly the same as those in Table I except for the probabilities. This is so because Unit 2 in the down state has zero loadable capacity, therefore resulting in a zero change in flow and zero change in unsupplied load. The new state probabilities are obtained by multiplying the probabilities of the states in Table I with the probability of Unit 2 being down.

With Unit 2 up, 150 MW additional loadable capacity is available. Combination of this with State 1 in Table I results in State 3 in Table II. In this state 60 MW of additional load can be supplied in Area B and 90 MW of additional loads can be supplied in Area A. As Unit 2 is in Area B, this results in a tie flow change towards A of 90 MW. This added to the tie flow value of State 1 in Table I results in a 90 MW flow towards Area A. The supplied load change of 90 MW and 60 MW in Areas A and B respectively, when superimposed on unsupplied loads of 180 MW in Area A and 120 MW in Area B, results in remaining unsupplied loads of 90 MW in Area A and 60 MW in Area B. Combination of Unit 2 up with State 2 in Table I results in State 4 in Table II. In this state only 100 MW capacity utilization of Unit 2 is possible because that is all that is required to satisfy the load which remains to be supplied. This results in zero remaining unsupplied load in both areas, and in a flow change of 60 MW towards Area A. This flow change superimposed on the existing partial flow of 80 MW towards Area B in State 2 in Table I, results in a new flow of 20 MW towards Area B.

The mechanics of computation in processing additional units in Tables III through VI is hoped now to be clear. What remains to be explained is the algorithm used to reduce the states to equivalent ones to prevent their number to grow excessively. This is done in the following two ways.

[1] By recognizing tie flow states which are not affected by subsequent unit additions. As these are therefore final states, they need not be addressed in the remaining computation.

[2] By combining generation states which are equal or sufficiently similar in terms of tie flows and unsupplied loads.

Both of the above are illustrated below.

In Table II, State 4 has zero unsupplied load. This means that any of the units as yet to be added, when combined with this state would not be loaded even if they were to be in their up state, as these units have more expensive power production costs than the units already loaded. Therefore, if State 4 in Table II were to be left interacting with the remaining Units 3 to 6, the states that would emanate from it would all be the same in terms of line flows and unsupplied loads. The combination of all these states would result in a state exactly the same as State 4 in Table II. Therefore this state can be extracted from Table II and saved in Table VII. The states in this table are combined with the final states obtained in Table VI after adding the last unit, namely Unit 6. As a result of this extraction, only the first 3 states in Table II are used in conjunction with Unit 3 to obtain Table III.

It is noted that State 5 in Table III, States 7 and 10 in Table IV, and State 10 in Table V, are all final states since they have zero unsupplied load and are removed as soon as encountered from subsequent computation and stored in Table VII.

To illustrate the combination concepts used, it was assumed that before addition of a next unit, the states to be processed need to be six or less to remain within acceptable computer requirements. In Table IV, after extraction of States 7 and 10, there are still 8 states remaining and accordingly, these need to be reduced to 6 states or less. The reduction is accomplished by combining state pairs labelled A, B, and C in Table IV. It is noted that both states in each pair are exactly the same in terms of tie flow and unsupplied load, and this fact makes the combination valid.

The combination of two equal states results in a state equal in terms of F_{AB}, UL_A and UL_B, and with a probability which is the sum of the probabilities of the two states. The equivalencing of the state description in terms of loading and spare on units is done by replacing these values by their weighted average.

In Table V, again state reduction is required. In this case no equal states are present. Therefore the most closely similar states are selected. Three pairs are selected and these are labelled A, B and C in Table V. The validity of the combination basically rests on the fact that if two equal states can be combined, so can two which for practical purposes are essentially equal. The final results obtained after having added all the units are tabulated in Table VIII.

3.0 Overload Minimization by Minimum Cost Departure from Optimal Economic Dispatch

The results in Table VIII contain a number of flows which exceed the maximum tie capacity of 80 MW. Some component of these overloads are due to the strict adherence to the optimal economic dispatch of the units. These overload components need to be eliminated before computing the load curtailments due to the tie capacity as implied by tie overloads. This is in accordance with the operating policy that load will not be curtailed to adhere to optimal economic dispatch.

The overloads in Table VIII which coexist with larger than zero unsupplied load due to generation cannot be minimized or eliminated by departing from optimal economic dispatch. The reason for this is that all generation capacity in the state resulting in the overload is already fully loaded, and therefore there is no spare capacity to permit rescheduling. Accordingly, the overload in States 2, 13, 14 and 15 in Table VIII are not due to adherence to the optimal economic dispatch. The overload in States 1 and 3, however, can be eliminated by modifying unit dispatch.

The unit loadings which results in the tie overload for State 1 is recorded in correspondence with this state on the right portion of Table VIII. From this information it is noted that in State 1, Units 5 and 6 are not loaded, but have unloaded capacity. As the overload is towards Area B, if either or both of Units 5 and 6 are in Area B, the overload can be decreased or eliminated depending on the available unloaded capacity on these units. Reference to Figure 3 shows that Unit 5 is in Area A while Unit 6 is in Area B. Therefore, only Unit 6 can be used to relieve the overload. It is clear that if Unit 6 is loaded to 40 MW and Units 4 and 3 are unloaded by an equal amount, the tie flow becomes 80 MW and therefore acceptable. The new flow and unit loadings now describing State 1 are given in Table IX. Since Unit 6 is more expensive than Units 3 and 4, there will be an increase in fuel costs. These are attributable to the tie capacity limitation. The expression for the computation of these fuel costs increase is given in Table IX.

Following a similar reasoning the overload for State 3 in Table VIII is also eliminated again with an increase in fuel costs. The modified State 3 and resulting fuel costs increase are also given in Table IX.

The summation of the fuel cost increases in Table IX represents the restriction in terms of fuel costs imposed by the tie capacity of 80 MW on the optimal dispatch of the generating units. For the purpose of simplifying the presentation of the concepts involved, the above increase in fuel costs to eliminate overloads was computed using the equivalent state descriptions in terms of loading and spare on the units as given in Table VIII. The use of these equivalents introduces an error. Figure 4.0 illustrates the algorithm for accurate computation of fuel cost penalties due to tie capacity for State 3 in Table VII. In the sample computation illustrated in Tables I through VIII, States 2 and 3 in Table VII were equivalenced and entered in Table VIII as State 1. The equivalencing was done after adding to these states the units not as yet considered, namely, Units 4, 5 and 6 for State 2 and Units 5 and 6 for State 3.

The processing illustrated in Figure 4 would proceed as follows. Following a unit addition, if there is a final state with an overload, for example State 7 in Table IV, this state is not stored in Table VII but is processed as shown in Figure 4 and States M and N in this figure would be stored instead in Table VII.

The fuel cost penalty incurred would be accumulated in a separate table. The expression for the fuel cost penalties due to States 2 and 3 processed as illustrated in Figure 4 is given below. Comparison of this with that computed for State 1 in Table IX shows the nature of the error which is made by using state equivalents.

Accurate Fuel Cost Increase for States 2 and 3 in Table VII =

$$= (.0918)(T)\ [(40.0)C_6 - (40.0)C_3] + (.01458)(T)[(40.0)C_6 - (40.0)C_4]$$

where: T = study period in hours
C_i = production cost in \$/Mwh for unit i.

4.0 Overload Minimization by Redistribution of Unsupplied Load Due to Generation

The remaining overloads in Table VIII, namely States 2, 13, 14 and 15, cannot be relieved by generation rescheduling as noted above. They can, however, be relieved by modifying the policy related to allocation of curtailment of load due to generation insufficiency. The policy used in obtaining the results in Table VIII assumed allocation of this curtailment proportionally to the total area loads. If it is assumed that this policy is applied within the constraints imposed by tie limitation, then the overloads of the above mentioned states can be eliminated, or minimized. For example, overload in State 15 in Table VIII can be eliminated by curtailing all of the unsupplied load due to generation in Area A. This results in UL_A = 100.0 MW, UL_B = and 0.0 MW, FAB =-80.0. Clearly, the tie capacity prevents the "fair" treatment of the customers in the two areas. Table XII gives the additional load curtailment burden, which the two areas have to bear due to tie limitation which forces redistribution of load curtailment due to generation shortage.

Table X gives the results of Table VIII modified by minimizing the overloads as far as possible by either generation rescheduling or by redistribution of unsupplied load due to generation shortage.

5.0 Load Curtailment due to Tie Capacity Limitation

Table X now contains only one state with the tie overloaded, namely State 2. The only way to relieve this overload is by curtailing 20.0 MW of load in Area B. Of course this results in a 20 MW unloading of the most expensive loaded generation in Area A. Therefore, to relieve overload in State 2, Area B suffers an additional load cut of 20.0 MW while Area A experiences a fuel cost saving due to generation unloading. The new State 2 is given in Table XI together with the expression to compute the fuel cost savings.

6.0 Recognition of Tie Line Failure Probability

To illustrate how tie line failure probability can be taken into account assume that the tie is made of two circuits each with a maximum load carrying capacity of 40 MW and each with failure probability of 0.01. Assuming independent failure behaviour of the circuits the tie can exist in three different transmission capacity states as follows:

State Number	State Description	Transmission Capacity in MW	Probability
1	both circuits in-service	80	0.9801
2	either circuits out-of-service	40	0.0198
3	both circuits out-of-service	0	0.0001

If it is assumed that the generation and load states are statistically independent of the tie states, which is a good assumption, then each of the three tie states can coexist with each of the load flow states in Table X. The analysis is then done by separately combining each of the three transmission states with the states in Table X and combining the results weighted by the probability of the tie state occurring. Of course alternatively the computation illustrated in Tables I through XII can be done three times: once taking the tie capacity to be 80 MW, once taking the tie capacity to be 40 MW; and once taking the tie capacity to be zero MW. The results would be combined by weighting them by the probability of tie capacity state used.

7.0 Modelling of Load and Hydraulic Generation

The analysis illustrated above was done assuming a single load level of 180 MW in Area A correlated with a single load level of 120 MW in Area B. A multi-level load model is straightforward to implement. The loads in the two areas would be represented as a series of correlated load level pairs, each with an assigned probability of occurrence. The analysis illustrated above would be repeated for each load level pair. Results from each load level pair would be combined by weighting them by the respective probability of occurrence of the load pair. The loads in the two areas could be considered independent without any additional complexity. However, such a load model is not considered realistic and is therefore not recommended.

The modelling of the hydraulic generation is complicated by the fact that the amount of water available for generation cannot be considered infinite. As a result, depending on how the water is used, it can happen that at some times during a typical day, although the hydraulic units are operable, there would be no water to generate the power. The technique described above can easily accept as input data the way in which the hydraulic generation would be operated for the typical day used for the analysis.

The load and hydraulic generation model is illustrated in Figure 5. The analysis described in Sections 2.0 through 6.0 would be repeated for each of the correlated set a through i in Figure 5. The results obtained with each such set would be weighted by the probability of the set occurring and combined.

8.0 Discussion of Results

With reference to Tables VIII through XII, the technique presented here permits computation of:

1. Probability density function of tie flows for:
   (a) generation dispatched in the most economical way assuming that the tie imposes no restrictions.
   (b) generation dispatched in the most economical way within the constraint imposed by tie line capacity and assuming a perfectly reliable tie.
   (c) Same as in (b) but recognizing tie elements failure probability.

2. Joint probability density function of load curtailment in the two areas separated into the following components:
   (a) Due to generation failure probability.
   (b) Due to inability to apply policy related to load curtailment due to generation because of tie restriction arising from its capacity limitation and failure performance.
   (c) Due to tie capacity limitation and failure performance.
3. Restriction imposed by the tie on optimal economic dispatch in terms of increased fuel costs.
4. Saving due to unused fuel as a result of load curtailment due to the tie.
5. Joint probability density function of unit loading with recognition of restrictions imposed on optimal economic dispatch by tie capacity and failure performance.

The usefulness of the above indices seems to be obvious especially for the comparison of alternative schemes for connecting two areas and will therefore not be explicitly discussed.

One of the current problems in the area of composite system reliability is that the curtailment due to generation and that due to transmission are computed separately. Therefore, their combination cannot be accomplished properly because:

(a) Some of the curtailment due to generation coexist with curtailment due to transmission. If this coexistence is not known the combination to obtain a resultant probability density of load curtailments cannot be done correctly.
(b) The distribution of load curtailments due to generation needs to be done in accordance with some operating policy. However, the application of this policy is restricted by transmission capabilities. Therefore, allocation to individual load buses of load curtailment due to generation shortage cannot be done without knowledge of the load flow state coexistent with generation shortages. The above of course also applies to load curtailment due to transmission.

The technique presented in this paper resolves the above problems by continually factoring into the analysis the restriction imposed by the transmission.

The use of the concepts presented to compute the joint probability density function of line flows was not explicitly discussed, but should be easy to visualize. For example for the problem solved, if the tie is composed of two equal circuits fully reliable, the flows on each circuit would be exactly half of those computed in Table VIII. Thus, the F_{AB} column in this table would be split into two columns, one for each of the circuits in the tie. Table VIII then would give the joint probability density function of the flows in the first and second circuits, the unsupplied load due to generation in the two areas, and the loadings and spares on each of the units.

9.0 Extrapolation of the Concepts Presented

The concepts illustrated above can be extrapolated with no further conceptual complexity for the:

(a) Recognition of undervoltages in the computation of load curtailments.
(b) Solution of the multi-area problem.
(c) Computation of fuel requirements with recognition of the restriction imposed by transmission limitations. The current techniques for computation of fuel requirements, essentially all based on reference (5) cannot recognize transmission limitations.

Also, the technique presented can be modified to compute the frequency and duration indices.

Work is now proceeding on developing the algorithms for the above extrapolation, and the results may be the subject of future papers. The computational burden for the above extrapolations will increase significantly, and, although it is expected to remain within practical limits, its magnitude remains as yet to be assessed.

10. Acknowledgements

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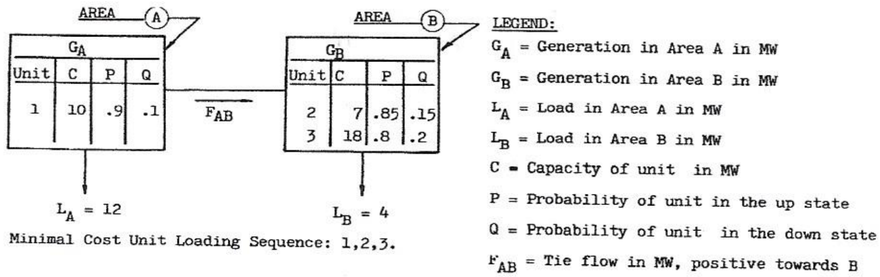


Figure 1 (a) - Sample system to illustrate computation of partial flows.

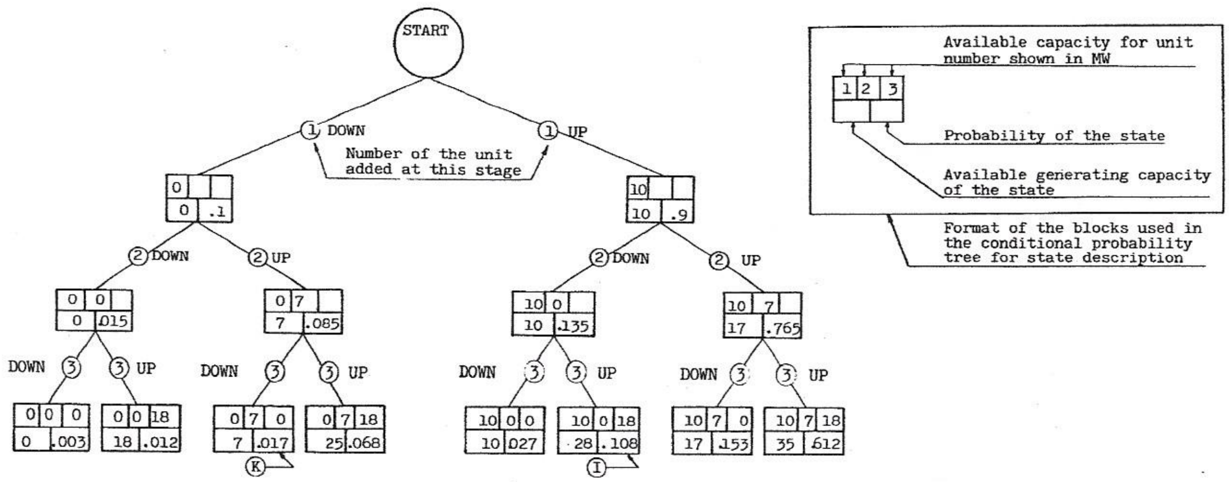
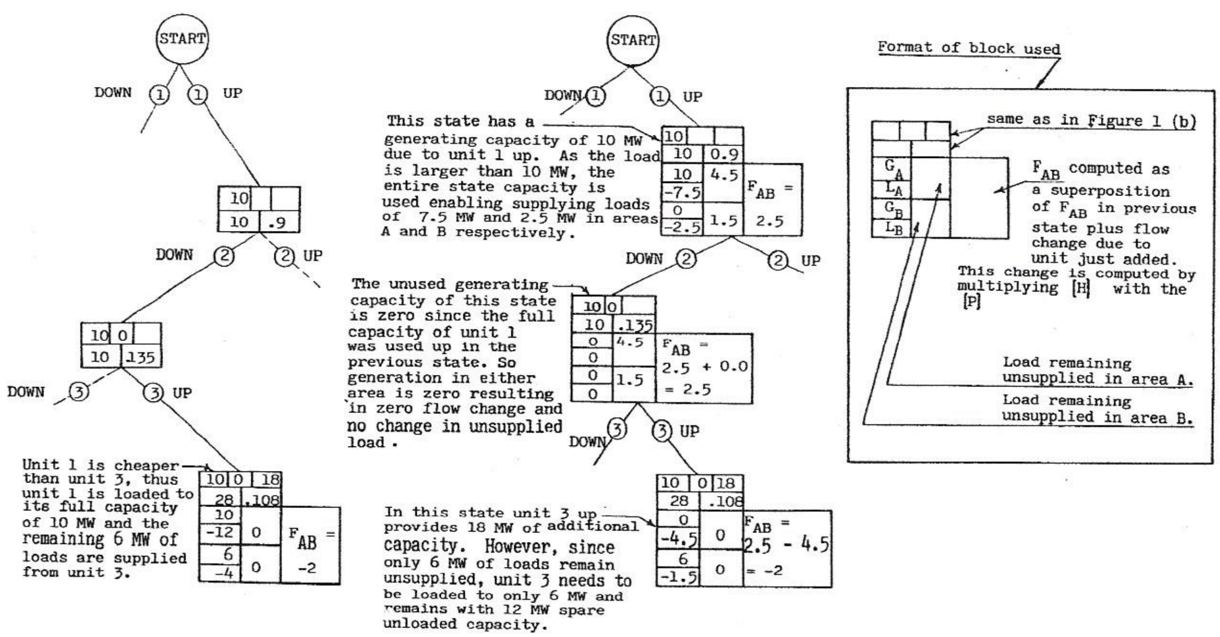


Figure 1 (b) - Conditional probability tree illustrating generation of all the possible states for the units in Figure 1 (a).



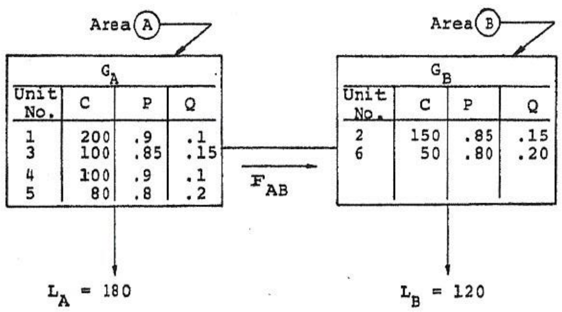
(a) Computation of the tie flow using final generation state I in Figure 1.

(b) Computation of tie flow by super position of flow changes after each unit addition.

Figure 2 - Illustration of the computation of tie flows by superposition of partial flows computed after each unit addition. The system solved is illustrated in Figure 1. For a two area problem, the tie flow can be computed directly once the load and generation in each area is known. For the sake of generality, the computation can be done using the expression $[F_e] = [H][P]$. In this problem this equation takes the form:

$$[F_{AB}] = [.5 \quad .5 \mid -.5 \quad -.5][G_A \quad L_A \mid G_B \quad L_B]^t \quad \text{where } t = \text{transpose}$$

Two nodes for each area were used to permit explicit separate accounting of generation and load in the two areas. Note that when there is insufficient generation to meet the load, the latter is assumed cut in the two areas in proportion to the respective area loads.



Maximum Tie Capacity = 80 MW
Minimal Cost Unit Loading sequence:
1,2,3,4,5,6.
Load cutting rule: Proportional to area loads.

Figure 3 - Description of system used to obtain the results in Tables I through XII. The legend defines the symbols used in this figure as well as those used in Tables I through XII.

LEGEND

G_A = Generation in Area A
G_B = Generation in Area B
C = Capacity of units in MW
P = Probability of unit in the up state. In the tables, this is the generation state probability
Q = Probability of unit in the down state
L_A = Load in Area A in MW
L_B = Load in Area B in MW
F_{AB} = Tie flow in MW, positive in direction A to B
UL_A = Unsupplied load in Area A
UL_B = Unsupplied load in Area B
L = Unit loading level
S = Spare unloaded unit capacity

State 3 From Table VII

F_{AB}	P	UL_A	UL_A	State Description in Terms of Unit Loadings											
				Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
				L	S	L	S	L	S	L	S	L	S	L	S
120	.018225	0.0	0.0	200.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0				

(6) Down (6) Up

F_{AB}	P	UL_A	UL_A	State Description in Terms of Unit Loadings											
				Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
				L	S	L	S	L	S	L	S	L	S	L	S
120	.003545	0.0	0.0	200.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0			0.0	0.0

Note that if there were another unloaded unit in Area B this, when up, would be used to relieve the overload. Thus in such case this state would be processed just as State 2 above.

F_{AB}	P	UL_A	UL_A	State Description in Terms of Unit Loadings											
				Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
				L	S	L	S	L	S	L	S	L	S	L	S
80.0	.01458	0.0	0.0	200.0	0.0	0.0	0.0	0.0	0.0	60.0	40.0			40.0	10.0

Figure 4 - Illustration of algorithm for accurate determination of minimum cost departure from optimal economic dispatch for overload relief for States 2 and 3 in Table VII.

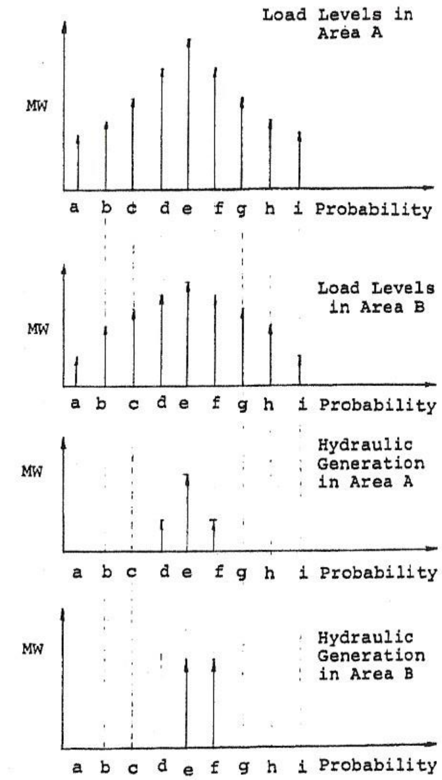


Figure 5 - Modelling of Load and Hydraulic Generation.

TABLE I

STATES AFTER ADDITION OF UNIT 1

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
Down	1	0.0	.1	180.0	120.0	0.0	0.0										
Up	2	80.0	.9	60.0	40.0	200.0	0.0										

TABLE II

STATES AFTER ADDITION OF UNIT 2 TO TABLE I

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
Down	1	0.0	.015	180.0	120.0	0.0	0.0	0.0	0.0								
	2	80.0	.135	60.0	40.0	200.0	0.0	0.0	0.0								
Up	3	-90.0	.085	90.0	60.0	0.0	0.0	150.0	0.0								
	4	20.0	.765	0.0	0.0	200.0	0.0	100.0	50.0								

TABLE III

STATES AFTER ADDITION OF UNIT 3 TO TABLE II FROM WHICH STATE 4

HAS BEEN REMOVED AND SAVED IN TABLE VII

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
Down	1	0.0	.00225	180.0	120.0	0.0	0.0	0.0	0.0	0.0	0.0						
	2	80.0	.02025	60.0	40.0	200.0	0.0	0.0	0.0	0.0	0.0						
	3	-90.0	.01275	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0						
Up	4	40.0	.01275	120.0	80.0	0.0	0.0	0.0	0.0	100.0	0.0						
	5	120.0	.11475	0.0	0.0	200.0	0.0	0.0	0.0	100.0	0.0						
	6	-50.0	.07225	30.0	20.0	0.0	0.0	150.0	0.0	100.0	0.0						

TABLE IV

STATES AFTER ADDITION OF UNIT 4 TO TABLE III FROM WHICH STATE 5

HAS BEEN REMOVED AND SAVED IN TABLE VII

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
Down	1	0.0	.00225	180.0	120.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
	2	80.0	.02025	60.0	40.0	200.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
	3 (B)	-90.0	.01275	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0				
	4	40.0	.01275	120.0	80.0	0.0	0.0	0.0	0.0	100.0	0.0	0.0	0.0				
	5 (A)	-50.0	.007225	30.0	20.0	0.0	0.0	150.0	0.0	100.0	0.0	0.0	0.0				
Up	6	40.0	.002025	120.0	80.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0				
	7 (C)	120.0	.018225	0.0	0.0	200.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0				
	8	-50.0	.011475	30.0	20.0	0.0	0.0	150.0	0.0	0.0	0.0	100.0	0.0				
	9	80.0	.011475	60.0	40.0	0.0	0.0	0.0	0.0	100.0	0.0	100.0	0.0				
	10	-30.0	.065025	0.0	0.0	0.0	0.0	150.0	0.0	100.0	0.0	50.0	50.0				

TABLE IV(a)

STATES IN TABLE IV AFTER COMBINATION OF STATE PAIRS A,B, AND C,

AND AFTER MOVING STATES 7 AND 10 TO TABLE VII

Table IV States Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
(1)	1	0.0	.000225	180.0	120.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
(2+9)	2	80.0	.0135	60.0	40.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0				
3	3	-90.0	.001275	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0				
(4+6)	4	40.0	.0033	120.0	80.0	0.0	0.0	0.0	0.0	38.636	0.0	61.364	0.0				
(5+8)	5	-50.0	.0187	30.0	20.0	0.0	0.0	150.0	0.0	38.636	0.0	61.364	0.0				

TABLE V

STATES AFTER ADDITION OF UNIT 5 TO THE STATES IN TABLE IV(a)

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
Down	1	0.0	.000045	180.0	120.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
	2	80.0	.0027	60.0	40.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	0.0	0.0		
	3 (A)	-90.0	.00255	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
	4	40.0	.00066	120.0	80.0	0.0	0.0	0.0	0.0	38.636	0.0	61.364	0.0	0.0	0.0		
	5 (B)	-50.0	.00374	30.0	20.0	0.0	0.0	150.0	0.0	38.636	0.0	61.364	0.0	0.0	0.0		
Up	6	32.0	.00018	132.0	88.0	0.0	0.0	0.0	0.0	0.0.	0.0	0.0	0.0	80.0	0.0		
	7 (C)	112.0	.0108	12.0	8.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	80.0	0.0		
	8	-58.0	.00102	42.0	28.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	80.0	0.0		
	9	72.0	.00264	72.0	48.0	0.0	0.0	0.0	0.0	38.636	0.0	61.364	0.0	80.0	0.0		
	10	-30.0	.01496	0.0	0.0	0.0	0.0	150.0	0.0	38.636	0.0	61.364	0.0	50.0	30.0		

TABLE V(a)

STATES IN TABLE V AFTER COMBINATION OF STATE PAIRS A, B, AND C,

AND AFTER MOVING STATE 10 TO TABLE VII

Table V States Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
(1)	1	0.0	.000045	180.0	120.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
(2+9)	2	76.045	.00534	65.933	43.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0		
(3)	3	-90.0	.00255	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
(4+6)	4	38.286	.00084	122.571	81.714	.0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0		
(5+8)	5	-51.714	.00476	32.571	21.714	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0		
(7)	6	112.0	.0108	12.0	8.0	30.0	0.0	0.0	0.0	85.0.	0.0	85.0	0.0	80.0	0.0		

TABLE VI

STATES AFTER ADDITION OF UNIT 8 TO STATES IN TABLE V(a)

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Generation State Description in Terms of Loading and Spare on Units											
						Unit 1		Unit 2		Unit 3		Unit 4		Unit 5		Unit 6	
						L	S	L	S	L	S	L	S	L	S	L	S
Down	1	0.0	.000009	180.0	120.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	76.045	.001068	65.933	43.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0	0.0	0.0
	3	-90.0	.000051	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	4	38.286	.000168	122.571	81.714	0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	0.0	0.0
	5	-51.714	.000952	32.571	21.714	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	0.0	0.0
	6	112.0	.00216	12.0	8.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	80.0	0.0	0.0	0.0
Up	7	-30.0	.000036	150.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0
	8	46.045	.004272	35.933	23.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0	50.0	0.0
	9	-120.0	.000204	60.0	40.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0
	10	8.286	.000672	92.571	61.714	0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	50.0	0.0
	11	-81.714	.003808	2.571	1.714	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	50.0	0.0
	12	100.0	.00864	0.0	0.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	80.0	0.0	20.0	30.0

TABLE VII

FINAL STATES EXTRACTED DURING COMPUTATION

From Step	State No.	F_{AB}	P	UL_A	UL_B	Unit 1 L	Unit 1 S	Unit 2 L	Unit 2 S	Unit 3 L	Unit 3 S	Unit 4 L	Unit 4 S	Unit 5 L	Unit 5 S	Unit 6 L	Unit 6 S
						Generation State Description in Terms of Loading and Spare on Units											
2	1	20.0	.765	0.0	0.0	200.0	0.0	100.0	50.0								
3 (A)	2	120.0	.11475	0.0	0.0	200.0	0.0	0.0	0.0	100.0	0.0						
4 (A)	3	120.0	.018225	0.0	0.0	200.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0				
4 (B)	4	-30.0	.065025	0.0	0.0	0.0	0.0	150.0	0.0	100.0	0.0	50.0	50.0				
6 (B)	5	-30.0	.01496	0.0	0.0	0.0	0.0	150.0	0.0	38.636	0.0	61.364	0.0	50.0	30.0		

TABLE VIII

FINAL RESULTS: THESE ARE THE STATES IN TABLE VII WITH PAIRS A AND B COMBINED, PLUS THE STATES IN TABLE VI RENUMBERED TO HAVE FLOWS IN DESCENDING ORDER

State of Unit Added	State No.	F_{AB}	P	UL_A	UL_B	Unit 1 L	Unit 1 S	Unit 2 L	Unit 2 S	Unit 3 L	Unit 3 S	Unit 4 L	Unit 4 S	Unit 5 L	Unit 5 S	Unit 6 L	Unit 6 S
						Generation State Description in Terms of Loading and Spare on Units											
	1	120.0	.132975	0.0	0.0	200.0	0.0	0.0	0.0	86.29	0.0	13.71	77.66	0.0	64.0	0.0	40.0
	2	112.0	.00216	12.0	8.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	80.0	0.0	0.0	0.0
	3	100.0	.00864	0.0	0.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	80.0	0.0	20.0	30.0
	4	76.045	.001068	65.933	43.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0	0.0	0.0
	5	46.045	.004272	35.933	23.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0	50.0	0.0
	6	38.286	.000168	122.571	81.714	0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	0.0	0.0
	7	20.0	.765	0.0	0.0	200.0	0.0	100.0	50.0	0.0	85.0	0.0	90.0	0.0	64.0	0.0	40.0
	8	8.286	.000672	92.571	61.714	0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	50.0	0.0
	9	0.0	.000009	180.0	120.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	10	-30.0	.079985	0.0	0.0	0.0	0.0	150.0	0.0	88.523	0.0	52.125	40.648	9.352	57.64	0.0	40.0
	11	-30.0	.000036	150.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0
	12	-51.714	.000952	32.571	21.714	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	0.0	0.0
	13	-81.714	.003808	2.571	1.714	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	50.0	0.0
	14	-90.0	.000051	90.0	60.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	15	-120.0	.000204	60.0	40.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0

TABLE IX

TIE OVERLOAD ELIMINATION BY MINIMUM COST DEPARTURE FROM THE OPTIMAL ECONOMIC DISPATCH FOR STATES 1 and 3 IN TABLE VIII

State From Table VIII	F_{AB}	P	UL_A	UL_B	Unit 1 L	Unit 1 S	Unit 2 L	Unit 2 S	Unit 3 L	Unit 3 S	Unit 4 L	Unit 4 S	Unit 5 L	Unit 5 S	Unit 6 L	Unit 6 S
					Generation State Description in Terms of Loading and Spare on Units											
1	80.0	.132975	0.0	0.0	200.0	0.0	0.0	0.0	60.0	26.29	0.0	91.37	0.0	64.0	40.0	0.0
3	80	.000864	0.0	0.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	60.0	20.0	40.0	10.0

Fuel Cost Increase for State 1 = (Cost of Unit 6 Loading Increase) - (Saving due to unloading of units 3 and 4)
$= \$(40)(.132975)(T)(C_6) - (13.71)(.132975)(T)(C_4) - (26.29)(132975)(T)(C_3)$
$= \$(.132975)(T)[(40.0)\ C_6 - (13.71)\ C_4 - (26.29)\ C_3]$

Fuel Cost Increase for State 3 = (Cost of Unit 6 Loading Increase) - (Saving due to loading of Unit 5)
$= \$\ (.00864)(T)[(20.0)\ C_6 - (20.0)\ C_5]$

In above two expressions:

T = study period in hours
C_i = fuel cost in \$/MWh for unit i

TABLE X

THIS TABLE IS OBTAINED FROM TABLE VIII BY REPLACING STATES 1 AND 3 WITH THOSE FROM TABLE IX AND BY MINIMIZING OVERLOADS BY REDISTRIBUTION OF UNSUPPLIED LOAD DUE TO GENERATION INSUFFICIENCY

State No.	F_{AB}	P	UL_A	UL_B	Unit 1 L	Unit 1 S	Unit 2 L	Unit 2 S	Unit 3 L	Unit 3 S	Unit 4 L	Unit 4 S	Unit 5 L	Unit 5 S	Unit 6 L	Unit 6 S
					Generation State Description in Terms of Loading and Spare on Units											
1	80.0	.132975	0.0	0.0	200.0	0.0	0.0	0.0	60.0	26.29	0.0	91.37	0.0	64.0	40.0	0.0
2	100.0	.00216	0.0	20.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	80.0	0.0	0.0	0.0
3	80.0	.00864	0.0	0.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	60.0	20.0	40.0	10.0
4	76.045	.001068	65.933	43.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0	0.0	0.0
5	46.045	.004272	35.933	23.955	15.169	0.0	0.0	0.0	62.078	0.0	73.315	0.0	39.551	0.0	50.0	0.0
6	38.286	.000168	122.571	81.714	0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	0.0	0.0
7	20.0	.765	0.0	0.0	200.0	0.0	100.0	50.0	0.0	85.0	0.0	90.0	0.0	64.0	0.0	40.0
8	8.286	.000672	92.571	61.714	0.0	0.0	0.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	50.0	0.0
9	0.0	.000009	180.0	120.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	-30.0	.079985	0.0	0.0	0.0	0.0	150.0	0.0	88.523	0.0	52.125	40.648	9.352	57.64	0.0	40.0
11	-30.0	.000036	150.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0
12	-51.714	.000952	32.571	21.714	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	0.0	0.0
13	-80.0	.003808	4.285	0.0	0.0	0.0	150.0	0.0	30.357	0.0	48.215	0.0	17.143	0.0	50.0	0.0
14	-80.0	.000051	100.0	50.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	-80.0	.000204	100.0	0.0	0.0	0.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0

TABLE XI

TIE OVERLOAD ELIMINATION BY LOAD CURTAILMENT FOR STATE 2 IN TABLE X

State From Table X	F_{AB}	P	UL_A	UL_B	Unit 1 L	Unit 1 S	Unit 2 L	Unit 2 S	Unit 3 L	Unit 3 S	Unit 4 L	Unit 4 S	Unit 5 L	Unit 5 S	Unit 6 L	Unit 6 S
					Generation State Description in Terms of Loading and Spare on Units											
2	80.0	.00216	0.0	40.0	30.0	0.0	0.0	0.0	85.0	0.0	85.0	0.0	60.0	20.0	0.0	0.0

Fuel Cost Decrease for State 2 = Saving due to unloading of Unit 5
$= \$\ (.00216)\ (T)\ (C_5)\ (20.0)$

where T = study period in hours
C_i = production cost of unit i in \$/Mwh

TABLE XII

LOAD CURTAILMENT ATTRIBUTABLE TO CAPACITY SHORTAGE OF THE TIE

State No. From Table VIII	Additional Load Curtailment in MW Forced on the Areas as a Result of restriction imposed by Tie Capacity on Allocation of Load Curtailment due to Generation Shortage in Accordance with Operating Policy		Decrease in Load Curtailment in MW Forced on the Areas as a Result of Restriction imposed by Tie Capacity on Allocation of Load Curtailment due to Generation Shortage in Accordance with Operating Policy		Load Curtailment in MW Due to Tie Capacity Limitation		Probability
	Area A	Area B	Area A	Area B	Area A	Area B	
2		12.0	12.0			20	.00216
13	1.714			1.714			.003808
14	10.0			10.0			.00005
15	40.0			40.0			.00204