Calculation of Power Systems Inertia and Frequency Response

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***Abstract*—Fundamental changes are taking place in the way that electricity is produced and consumed. These changes are raising concerns related to the deterioration of Inertial and Frequency responses . This paper presents a method to calculate the value of these responses using the data from system disturbances that result in sudden loss of generation. The method has very modest data and computational requirements and has a number of practical applications such as monitoring the deterioration of these responses over time. The method is illustrated by applying it to an actual generation loss incident on the Eastern Interconnection.**

***Index Terms*—Frequency Response, Inertia, Inertial Response, Angular Momentum, Governor Frequency Response, Equipment Frequency Response, System Disturbance, Frequency Deviation, Droop, Generation Loss, Self Regulation.**

I. Introduction

Frequency Response (FR) and Inertia (M) on our interconnections are declining [1]–[4]. This trend is considered to be due to emerging new generation technologies, new load supply technologies, and new types of loads. As FR and M are of fundamental importance to the reliability of our interconnections, the North American Electric Reliability Corporation (NERC) has initiatives [1], [2] to study and monitor these parameters. Early last year the Federal Energy Regulatory Commission (FERC) also acted on this issue and revised the pro forma for large and small generator interconnection agreements to require these facility to have adequate frequency response capability. The effective monitoring of FR and M requires a power system simulation model which does not require large amounts of data, does not have large computing requirements, and can be applied to study several scenarios quickly to validate result correctness. This paper presents such a simulation model, and demonstrates it by applying it to the May 12, 2012 generation loss incident of 1711 MW on the Eastern Interconnection.

The authors are not aware of any method already proposed to calculate interconnection M and FR directly from frequency events data.

II. Frequency Response

FR is the megawatt change in the generation-load imbalance affected automatically by the power system itself due to frequency changes alone [5]. FR derives from the intrinsic frequency characteristics of power system equipment (EFR) and from the governors of the individual generators (GFR). These two responses are fundamentally different.

EFR is deployed without delay immediately as soon as the frequency starts to change. Its magnitude is a result of equipment physics and design practices. Not all equipment provide beneficial EFR [6], [7]. To be beneficial EFR has to oppose frequency change.

GFR starts to be deployed with a time delay of about 2 to 5 seconds after the frequency starts to change and it is fully deployed by about 2 to 4 seconds later. This delay is due to governors dead bands, time constants, and gates or valves velocity limits imposed to avoid equipment damage. Its magnitude is a result of governor Droop settings and of generator loading practices and, as a result, it is somewhat unpredictable.

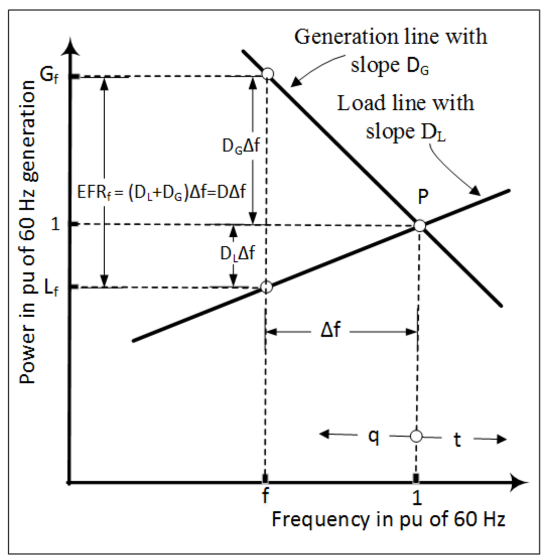

Fig. 1. Frequency response diagram for a system which has beneficial EFR, namely, an EFR that opposes the frequency change. All quantities represent magnitudes.

III. Equipment Frequency Response Model

The impact of EFR on the system is illustrated in Fig. 1. In the vicinity of 60 Hz it is reasonable to represent these impacts

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as straight lines. The generation line represents the aggregate FR of all generation equipment. For this FR to be beneficial, the generation line must have a negative slope, as only with such slope frequency changes are opposed by the resulting generation changes. The Load line represents the aggregate FR of all load equipment. For this FR to be beneficial the load line must have a positive slope as only with such slope frequency changes are opposed by the resulting load changes. Fig. 1 introduces the quantity D which is related to the slope of the generation and load lines. This quantity, which determines the magnitude of the EFR for a given frequency change, is referred to herein as System Self Regulation.

It is of interest to note from Fig. 1 that a system has beneficial EFR if $\underline{D}_L > \underline{D}_G$, where the underline denotes magnitude and sign. Systems with this characteristic have stable operating points such as P. In fact, frequency perturbations in the q or t direction result in imbalances of a sign that accelerates the system back to P. Systems where $\underline{D}_L \leq \underline{D}_G$ do not have stable operating points. This can be shown from Fig. 1 by changing the slopes of the lines. Further, Fig. 1 shows that the magnitude of the System Self Regulation D is given by the summation of the magnitudes of the generation and of the load self regulations.

The frequency response in a system that has suffered a generation loss equal to ΔP is illustrated in Fig. 2. In the analysis of disturbances such as this, it is convenient to express values in per unit (pu) of P_0 and f_0 [8]. These bases are defined in Fig. 2 which shows all quantities in pu of these bases. This figure shows that, in the absence of any remedial action such as governor response or load shedding, the system will find a new operating equilibrium at point B due to the system self regulation D. Immediately following the disturbance the frequency starts decelerating towards this point and, as frequency decreases, the accelerating power P_a decreases as a function of D according to eq. (1) below.

$$P_a = \Delta P - D\Delta f \tag{1}$$

Where:

- ΔP is the magnitude of generation loss in pu of P_0.
- Δf is the frequency deviation in pu of f_0.
- D is the system self regulation in pu of the torque of P_0 at f_0.

At point B, the accelerating power becomes zero and the frequency decay is arrested. This diagram shows that, in the absence of any remedial actions, the maximum possible value of the frequency deviation occurs at point B and it is given by $\Delta P/D$.

IV. Inertial Response Model

The rotating masses of the generators and motors on the system store kinetic energy. The amount of this stored energy is proportional to the system inertia M. Therefore, the larger the M the larger the kinetic energy stored. Following a disturbance that results in a generation loss, the frequency

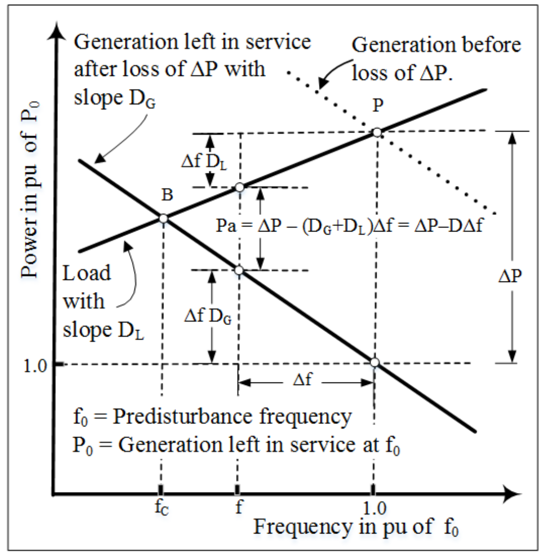

Fig. 2. Frequency response following a loss of generation ΔP. All quantities represent magnitudes.

starts to decay and this results in the release of the stored kinetic energy in the form of power which is referred to as Inertial Response. This response, consistent with D'Alambert's principle, is given by eq. (2), below [9].

$$M\frac{d\omega}{dt} = -P_a \tag{2}$$

Where:

- M is the system angular momentum (inertia) in J·s/rad.
- ω is the system angular speed in rad/s.
- P_a is the accelerating power. At ω_0, this equals the amount of generation lost which is the same as the excess load.
- ω_0 is $2\pi f_0$.

The expression on the left hand side of eq. (2) represents the Inertial Response power which, therefore, is proportional to M and the frequency rate of change. Accordingly, for a given value of P_a, the rate of change is directly proportional to P_a and inversely proportional to M. Thus, for a given generation loss, M determines how fast frequency can change following the loss. The higher the M the slower the rate of change. Therefore, higher values of M translate into more stable operation and, therefore, play a critical role in ensuring a stable transition from predisturbance to post disturbance conditions. Once the system has remained stable, the higher inertia lowers the frequency decay rate and thus results in more time available for the implementation of remedial actions such as governor response. Eq. (2) illustrates the fact that the excess load keeps being supplied by the Inertial Response until the new equilibrium point B is reached.

It is important to note that the Inertial Response contributed by M is not the same as EFR or GFR. The fundamental difference is as follows: these two responses together determine the value at which the frequency decay is arrested and impact the time that it takes to get there; M, however, only impacts the time that it takes to get there.

V. The Frequency Deviation Equation

By considering all rotating masses on the system aggregated into a single mass, it can be shown that, following a disturbance that results in a loss of generation ΔP, the frequency deviation from predisturbance frequency is given by eq. (3) below.

$$\Delta f(t) = (1 - e^{\frac{-D}{M}t})\frac{\Delta P}{D} \tag{3}$$

Where:

- t is the time in seconds from the start of the disturbance.
- Δf is the frequency deviation in pu of predisturbance frequency f_0.
- M is the system inertia in pu of the P_0 torque at f_0. This pu base gives M the dimension of seconds and makes it equal to $2H$ where H is the inertia constant of rotating machines [9].
- D is the magnitude of system self regulation in pu of the P_0 torque at f_0 .
- ΔP is the amount of generation lost in pu of P_0. For a generation loss, ΔP is negative.

This equation is derived by starting with eq. (2), substituting $\omega = \omega_0 - \Delta\omega$, dividing through by P_0, substituting for P_a the expression developed for it in Fig. 2, and solving the resulting differential equation for Δf.

Eq. (3) is used to calculate the frequency excursion curve for given generation loss incidents. This equation does not model GFR and, therefore, it cannot reproduce the frequency excursion beyond the GFR arrival time. The modeling of GFR is discussed in the following section.

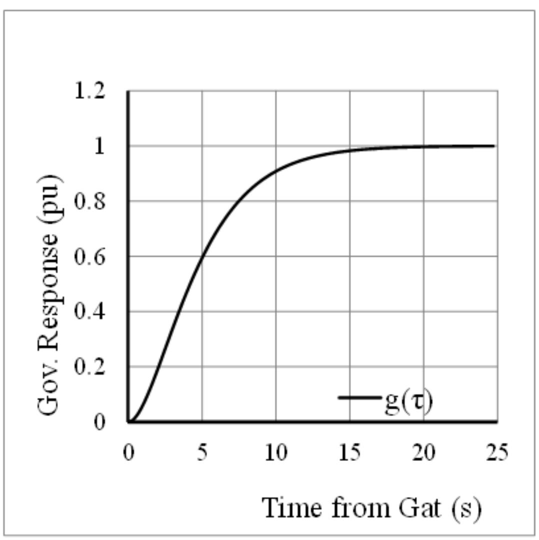

Fig. 3. Shape of the governor response function.

VI. Governor Frequency Response Model

Droop, expressed as a percentage of reference frequency, is defined as the frequency change required to have the governor make the output of the generator go from zero to full output. It can be shown that the magnitude of the additional power ΔG which is generated due to governor response is given by eq. (4) below.

$$\Delta G = K\Delta f \tag{4}$$

Where:

- ΔG is the amount of power from governors in pu of P_0.
- K is a constant given by $1/Edroop$, where $Edroop$ is the system effective Droop.
- Δf is the magnitude of frequency deviation in pu of f_0.

If all the generators remaining on the system after the disturbance are loaded with their governors set so that they can respond over the entire frequency excursion that the system experiences, and all the governors are set to the same Droop, then K would be equal to $1/Droop$. However, this is rarely, if ever, the case. In this paper the value of the Droop for the aggregate of all governors is referred to as effective Droop ($Edroop$).

A challenge with the modeling of GFR is that the full governor response magnitude is deployed on the system gradually with a response time which is due to a number of other factors besides the time constants of the governor themselves. In this paper the GFR at any time t is modeled as an incremental generation injection of a magnitude given by eq. (4) multiplied by a governor response function $g(\tau)$. These generation injections modify the initial conditions for calculating frequency deviations using eq. (3). Fig. 3 shows the shape of the governor response function. The time constants of this function are changed to match the frequency excursion of the disturbance that is being analyzed. Note that τ in this function is measured from the GFR arrival time. With this function the GFR injections are calculated with eq. (5) below.

$$GI(t) = \Delta G g(\tau) \tag{5}$$

Where:

- $GI(t)$ is the GFR injection in pu of P_0 at time t.
- $g(\tau)$ is the value of the governor response function at time τ.
- τ is $t - Gat$, where Gat is the time at which GFR arrives and t is the time measured from the start of the disturbance.

VII. Case Study

On May 12, 2012 the Eastern Interconnection suffered a disturbance that resulted in a generation loss of 1711 MW [2]. The steps to calculate M, D, $Edroop$, Gat, and $g(\tau)$ for the Eastern Interconnection at the time of this disturbance are described in the following subsections.

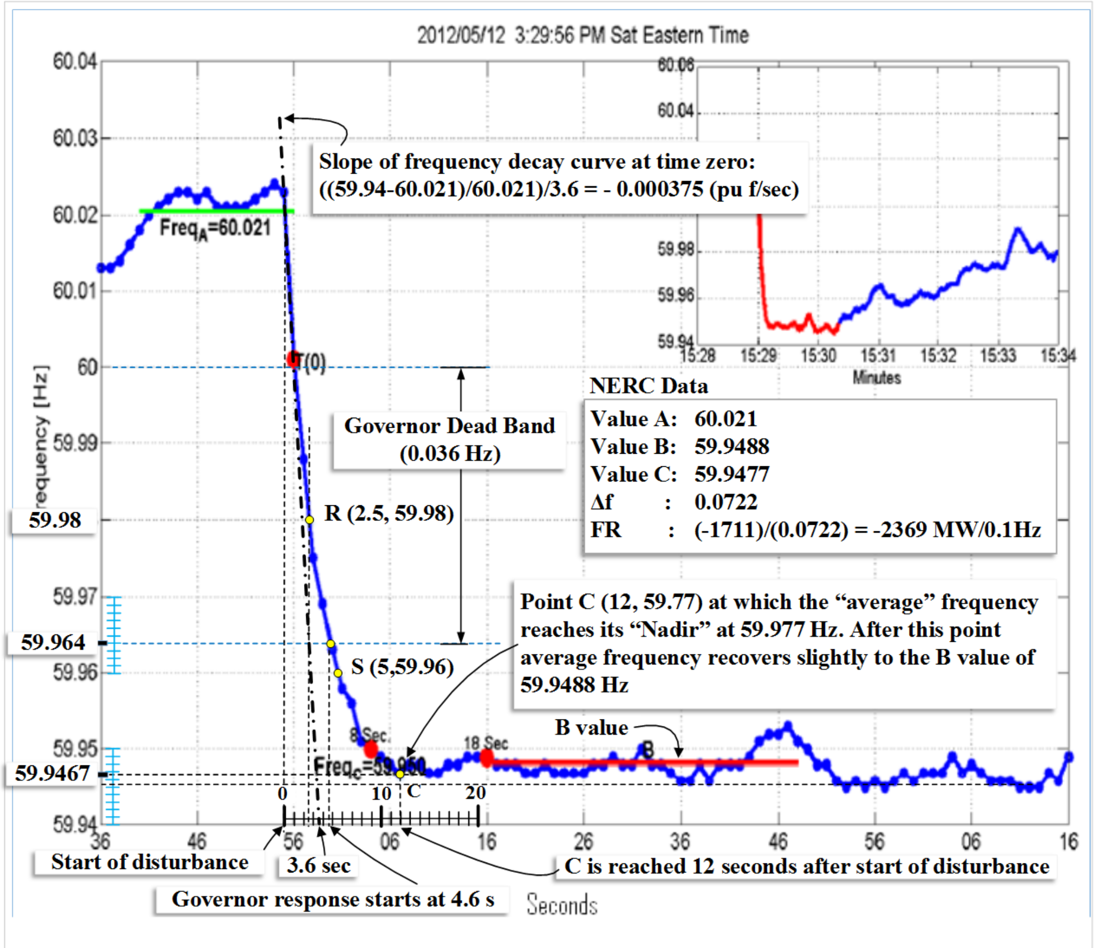

Fig. 4. Frequency excursion recorded following the loss of 1711 MW of generation on the Eastern Interconnection on May 12, 2012 [2].

A. Calculation of the pu value of the generation loss ΔP.

This calculation requires the generation left in service P_0 after the disturbance. This value was not available from the NERC report from which the data in Fig. 4 was obtained [2]. Based on historical load data, P_0 was estimated to be $335,000$ MW. With P_0 of $335,000$ MW, the pu value of the generation loss is given by eq. (6), below, where the negative sign denotes a generation deficiency.

$$\Delta P = \frac{-1711}{335000} = -0.005107 \text{ pu} \quad (6)$$

B. Calculation of the inertia M.

Differentiating eq. (3), setting t to zero and solving for M results in eq. (7) below.

$$M = \frac{\Delta P}{\Delta f_0'} \quad (7)$$

Where $\Delta f_0'$ is the frequency rate of change at $t = 0$ in pu of f_0.

The value for $\Delta f_0'$ can be obtained by fitting a tangent to the frequency excursion curve in Fig. 4 at time zero. The accuracy of this slope is essential to separate the initial impact of M from that of D. If the frequency excursion curve were to be available in electronic format, the accuracy of $\Delta f_0'$ can be improved significantly with regression analysis. From Fig. 4, $\Delta f_0' = -0.000375$. Now M can be calculated in eq. (8) below.

$$M = \frac{-0.005107}{-0.000375} = 13.63 \text{ s} \quad (8)$$

C. Calculation of the GFR arrival time Gat.

As shown in Fig. 4, assuming that the governor dead band is 0.0036 Hz, the governor response arrival time is calculated to be 4.6 s.

D. Calculation of the system self regulation D.

Point R in Fig. 2, with coordinates of $t = 2.5$ s and $f = 59.98$ Hz, occurs at a time at which the only frequency response present is from D, as the GFR arrives at 4.6 s as determined in subsection VII-C. Substituting in eq. (3) ΔP from subsection VII-A, M from subsection VII-B, and the coordinates for point R, gives $D = 3.65$ pu.

E. Calculation of the system effective Droop Edroop.

At the time that the disturbance occurred, the system was operating at a frequency of 60.021 Hz. Therefore, with respect to 60 Hz balanced operation, the system had an excess of generation. The Δf by which the system frequency has to be reduced is given by eq. (9), below.

$$\begin{aligned}\Delta f &= \frac{60.021 - 60}{f_0} \\ &= 0.0003499 \text{ pu}\end{aligned} \tag{9}$$

Knowing from subsection VII-D that the system D is 3.65 pu, the amount ΔG in MW by which the predisturbance system generation had to be reduced for 60 Hz operation is given by eq. (10) below.

$$\begin{aligned}\Delta G &= 3.65 \times 0.0003499 \times 335000 \\ &= 427.84 \text{ MW}\end{aligned} \tag{10}$$

Therefore, with respect to 60 Hz operation, following the disturbance the system had a generation deficiency of $(1711 - 427.84) = 1283.16$ MW. As the governors operate with 60 Hz as a reference, the combined governor and equipment frequency response which was deployed to arrest the frequency at 59.9488 Hz, value B in Fig. 4, had to be equal to 1283.16 MW. Accordingly, the average combined self regulation from governors and equipment is given by eq. (11) below.

$$\begin{aligned}D_c &= \frac{(1283.16/335000)}{(60 - 59.9488)/60.021} \\ &= 4.49 \text{ pu}\end{aligned} \tag{11}$$

As the system self regulation D is known to be 3.65 pu from subsection VII-D, the average response from governors is given by eq. (12), below.

$$\begin{aligned}D_{gov} &= 4.49 - 3.65 \\ &= 0.84 \text{ pu}\end{aligned} \tag{12}$$

Dimensional inspection of eq. (4) shows that K is the same as D_{gov}. Therefore, the $Edroop$ displayed by the post-disturbance system is given by eq. (13) below.

$$\begin{aligned}Edroop &= \frac{1}{0.84} \\ &= 119.05\ \%\end{aligned} \tag{13}$$

F. Choosing the governor response function, $g(\tau)$.

To start the iterations to refine the parameters M, D, Gat, $Edroop$, and $g(\tau)$ itself, $g(\tau)$ was chosen with a time constant that gave full response in 5 seconds.

G. Refinement of parameters.

The calculations, for this step, were automated with the FREDEV program written by the authors. This program calculates the frequency curve based on eq. (3) and the GFR model discussed in section VI. Starting with the parameters for $g(\tau)$, ΔP, M, D, Gat, and $Edroop$ determined above, an iteration process is started in which the frequency curve is calculated and compared to that shown in Fig. 4. Based on this comparison, the above parameters are modified and the frequency curve is recalculated and the process is repeated until a good match with Fig. 4 is obtained. The frequency curve that this process converged to for this case is shown in Fig. 5. This figure also shows the final parameters used. Note that the coordinates of points A, R, S, C, and B in Fig. 4 and Fig. 5 match.

VIII. Result Analysis

As per Fig. 5 at the time of this disturbance the Eastern Interconnection had the following Inertia and FR characteristics: $M = 13.63$ s; $D = 3.65$ pu; $g(\tau)$ as shown in Fig. 3; $Edroop = 119.05$ %; $Gat = 3.9$ s.

The $Edroop$ of 119.05 % indicates poor governor response. Curve $G1$ and $G2$ in Fig. 6 confirms this conclusion. Curve $G2$ in this figure shows what the impact of an $Edroop$ of 5% would have been, and highlights the critical importance of governor response to frequency recovery and to keep the frequency above load shedding settings before the arrival of Automatic Generation Control (AGC) and manual operating actions. In terms of MW, from eq. (4), the $Edroop$ of 119.05 % contributed 240 MW towards the arresting of the frequency decay.

Curves $D1$ and $D2$ in Fig. 6 illustrate the importance of system self regulation D. In terms of MW, the D of 3.65 pu contributed 1471 MW towards the arresting of the frequency decay. So, in this case, the self regulation deserves most of the credit for arresting the frequency decay.

Curves $M1$ and $M2$ in Fig. 6 illustrate the impact of inertia M. The lower M results in a faster decay of the frequency, while the higher M results in a slower decay of the frequency. In both cases, however, the value at which the frequency is arrested does not change since, as discussed above, this arrested value is determined solely by the combined impact of self regulation and governors.

The Gat of 3.9 seconds considered together with the $g(\tau)$ in Fig. 3 indicate that the full deployment of governor response occurred 23.9 seconds after the start of the disturbance.

It can be shown that the inertia constant M for a system where the only rotating masses are contributed by generators is given by the weighted average of the inertia constants of the individual generators, with the generator ratings as the weighting factor. Accordingly, the system M calculated herein would be expected to be somewhat lower than the largest inertia constant exhibited by any of the individual generators. Based on this, the calculations herein would be expected to result in a system M of about 10 seconds. The fact that the M calculated is 13.63 seconds, indicates that the system has significant amounts of kinetic energy contributed by motors. As the M of the system due to generators only can be easily calculated by summing the M of the individual generators, the method presented can also be used to evaluate the inertia contributed by motors.

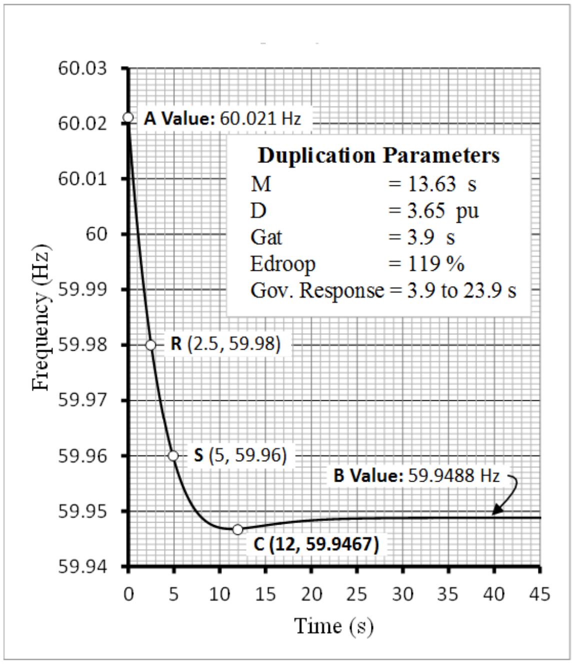

Fig. 5. Duplication of the frequency curve of Fig. 4.

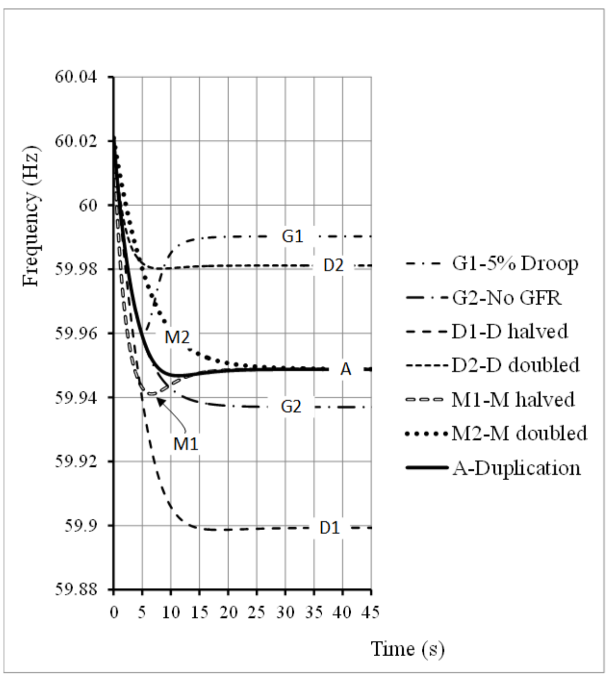

Fig. 6. A "what if" Analysis.

IX. Conclusion

This paper has presented a method to calculate Inertia M, System Self Regulation D, governor effective Droop ($Edroop$), governor response arrival time Gat, and governor response function $g(\tau)$ using data gathered for system disturbances that result in significant frequency excursions. The method has very modest data and computational requirements. The calculations were done using the FREDEV program developed by the authors. This program is considered useful to:

1. Monitor the deterioration of interconnection Inertia and Frequency Responses with increasing penetration of new generation technologies, new load supply technologies, and new loads.
2. Monitor the extent to which guidelines related to governor settings and generation loading practices are being followed.
3. Monitor the deterioration of Inertia from motors with increasing penetration of variable speed drives.
4. Justify and optimize the settings for existing or proposed under-frequency load shedding schemes.

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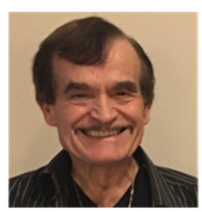

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